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EFFECT OF ELASTIC FOUNDATIONS ON DIVERGENCE AND FLUTTER OF AN ARTICULATED PIPE CONVEYING FLUID

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In this study, we generalize earlier investigations of Benjamin and Sugiyama & Païdoussis devoted to the stability of articulated pipes conveying fluid. The present study additionally incorporates the translational and rotational elastic foundations in an attempt to answer the following question: Do the elastic foundations increase the critical velocity of the fluid? It turns out that the attachment of the elastic foundation along the entire length of the pipe may either strengthen or weaken the system, with attendant increase or decrease in the critical velocity. The physical mechanism of the change of type of instability plays a crucial role in deciding whether or not the elastic foundation increases the critical velocity. If the elastic foundations are attached within the first pipe only, the instability mechanism is by flutter. If the elastic foundations are attached beyond the first pipe, then divergence may occur. The interplay of the two mechanisms may lead to a decrease of the critical velocity of the system with elastic foundations. A remarkable nonmonotonous dependence of the critical velocity with respect to the attachment foundation ratio is established.

1. INTRODUCTION

The vibrations and stability of pipes conveying fluid is a subject that attracted numerous investigators. The present state of the art is summarized in two monographs, by Chen (1987) and Païdoussis (1998), in addition to an extensive journal literature. Two review papers are of prime importance (Païdoussis 1987; Païdoussis & Li 1993) providing numerous relevant references and a critical overview of the subject. This study concentrates on the effect of elastic foundations on the stability behaviour of articulated pipes. The choice of the articulated pipes is made due to the fact that it is a two-degree-of-freedom system, whose analytical solution is tractable, and furthermore the model captures some of the characteristics of the continuous pipes conveying fluid.

The articulated pipes have been studied in several papers. These include the pioneering contribution by Benjamin (1961a), who studied the stability of a vertical cantilevered system of articulated pipes, showing that divergence is possible when gravity is taken into account

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and the fluid is sufficiently heavy. Païdoussis (1970) found that flexible vertical continuous pipes never experience divergence. Sugiyama (1981, 1983) demonstrated the stabilizing effect of damping on articulated pipes when a big mass is attached to the portion near the free end. Furthermore, he showed (Sugiyama 1984) that an additional spring support can destabilize the system and cause divergence-type instability. Sugiyama & Païdoussis (1982) elucidated that the critical velocity of a system of articulated pipes is strongly influenced by the relative length and mass per unit length of the constituting pipes.

An interesting tale of the effect of elastic foundation on non-conservative systems relates to work started in 1972, when the effect of the Winkler foundation on the stability of the so called Beck's column was studied by Smith & Herrmann (1972). They arrived at the unexpected conclusion that the elastic foundation did not increase the flutter load. Since there appears to be some mathematical correlation between the column subjected to the "follower" forces and the pipes conveying fluid, one may rightfully question the effect of the elastic foundation on the behaviour of the pipes. The effect of an elastic foundation on the fluid-conveying pipe was investigated in several studies. Stein & Tobriner (1970) considered the effect of internal pressure in the equation of motion and introduced a Winkler foundation to study the dynamic characteristics of a pipe of infinite length; in this case the foundation was necessary to guarantee the equilibrium of the system. Becker *et al.* (1978) illustrated the variety of behaviours under the introduction of Winkler and rotatory foundations acting alone or in concert along the entire length of a cantilevered continuous pipe. One of their notable results was the stabilizing effect of the Winkler foundation in the pipe conveying fluid, whereas in the model considered by Smith & Herrmann (1972) it has no influence; another interesting conclusion was the much stronger stabilizing effect of the rotatory foundation in contrast to that due to the Winkler foundation. Lottati & Kornecki (1986) derived numerous results for the case of varying fluid-over-total mass ratio when the Winkler foundation was present. Their study involved both cantilevered and clamped–clamped pipes. They showed that in both cases, the elastic foundation stabilizes the system. Other relevant papers include those by Roth & Christ (1962) and Roth (1964). There is a considerable body of literature on the effect of an elastic foundation in the realm of follower forces, but since these are outside the scope of the present investigation, they will not be touched upon. This study deals with articulated pipes on partial elastic foundations. Emphasis is placed on the question of whether the presence of an elastic foundation increases the critical velocity.

2. BASIC EQUATIONS

Consider a two-degree-of-freedom system consisting of two rigid pipes. The first pipe is connected to the wall through the massless viscoelastic joint A. The two pipes are connected with the joint B as depicted in Figure 1(a). The angles φ_1 and φ_2 uniquely determine the position of the structure. The first pipe has a length a, whereas that of the second pipe is b. Each pipe has a mass per unit length m_p. An incompressible fluid, with mass per unit length m_f, flows through the pipe with a constant velocity V. The two joints are assumed to have different restoring moment stiffnesses, R_A and R_B, as well as different damping coefficients, C_A and C_B, respectively. In this study, we generalize the analyses performed by Benjamin (1961a) and Sugiyama & Païdoussis (1982). We introduce a homogeneous elastic foundation attached to the pipe for a length l such that $0 \leq l \leq a + b$. The foundation is a combination of the Winkler foundation, with modulus k_w, and the rotatory foundation providing a restoring moment with modulus k_r.

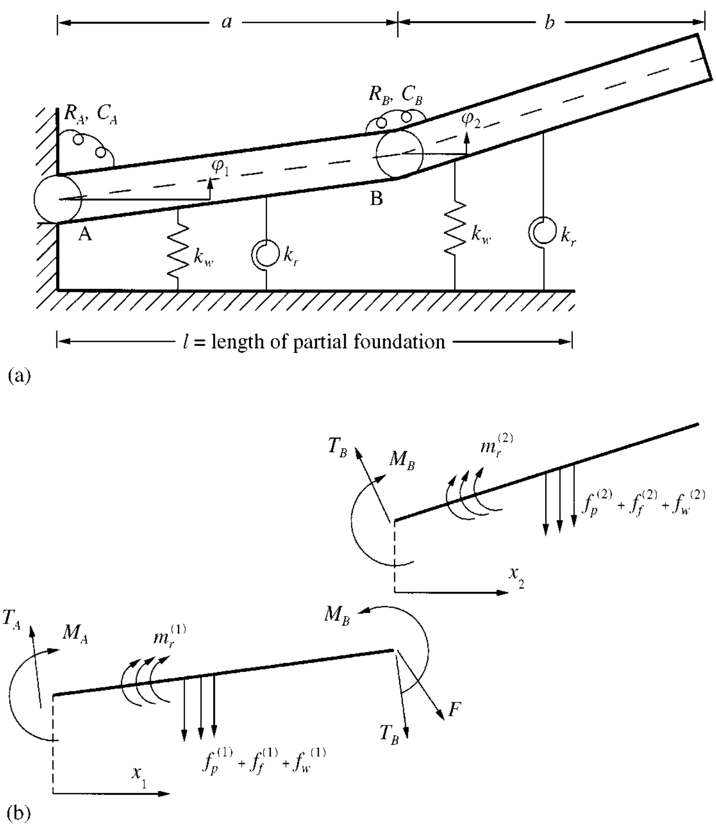

Figure 1. (a) Mathematical model of a two-degree-of-freedom articulated pipe conveying fluid. (b) A free body diagram.

2.1. Equation of Motion

The equation of motion for small vertical displacement of the articulated pipe will be derived here by considering the dynamic equilibrium of the system. The free-body diagram of each pipe is depicted in Figure 1(b). The first pipe is subjected to (i) the inertial forces of the pipe $f_p^{(1)}$ and of the fluid $f_f^{(1)}$; (ii) the translational forces $f_w^{(1)}$ exerted by the Winkler foundation; (iii) the restoring moment $m_r^{(1)}$ produced by the rotatory foundation; (iv) reaction T_A and moment M_A in the first joint; (v) reaction T_B and moment M_B in the second joint; and (vi) force F exerted at joint B by the fluid when the fluid changes its direction by the angle $\varphi_2 - \varphi_1$. Analogously, the second pipe is subjected to the inertial forces of the pipe, $f_p^{(2)}$, and of the fluid, $f_f^{(2)}$, the Winkler foundation force $f_w^{(2)}$, restoring moment $m_r^{(2)}$, and reactions T_B and M_B. We introduce the local coordinates, namely, x_1 in the first pipe and x_2 in the second pipe. The inertial forces of the pipes read

$$f_p^{(1)} = m_p \ddot{\varphi}_1 x_1, \quad f_p^{(2)} = m_p(\ddot{\varphi}_1 a + \ddot{\varphi}_2 x_2). \tag{1}$$

The inertial forces of the liquid are obtained by considering its acceleration relative to the fixed reference system. The vertical position of a fluid particle inside the first pipe with respect to the local coordinate system x_1 is given by $\varphi_1 x_1$, and its velocity by $\dot{\varphi}_1 x_1 + \varphi_1 V$, while the acceleration is $\ddot{\varphi}_1 x_1 + 2\dot{\varphi}_1 V$. Thus, the Coriolis acceleration $2\dot{\varphi}_1 V$ and the angular acceleration $\ddot{\varphi}_1 x_1$ are the only contributions in the first pipe. In the second pipe, the vertical position of a fluid particle with respect to the local coordinate system x_2 is given by $\varphi_1 a + \varphi_2 x_2$, so that, along with the Coriolis acceleration $2\dot{\varphi}_2 V$ and the angular acceleration $\ddot{\varphi}_2 x_2$, the translational acceleration $\ddot{\varphi}_1 a$ has to be included also; hence,

$$f_f^{(1)} = m_f(\ddot{\varphi}_1 x_1 + 2V\dot{\varphi}_1), \qquad f_f^{(2)} = m_f(\ddot{\varphi}_1 a + \ddot{\varphi}_2 x_2 + 2V\dot{\varphi}_2). \tag{2}$$

The forces exerted by the foundations have the following forms:

$$f_w^{(1)} = k_w \varphi_1 x_1, \quad f_w^{(2)} = k_w(\varphi_1 a + \varphi_2 x_2),$$
$$m_r^{(1)} = k_r \varphi_1, \quad m_r^{(2)} = k_r \varphi_2. \tag{3}$$

The moment reactions read

$$M_A = R_A \varphi_1 + C_A \dot{\varphi}_1, \qquad M_B = R_B(\varphi_2 - \varphi_1) + C_B(\dot{\varphi}_2 - \dot{\varphi}_1). \tag{4}$$

The force F is given by the time derivative of the momentum change of the fluid entering from the first pipe to the second one

$$F = \frac{\mathrm{d}}{\mathrm{d}t}(m_f V^2 \varphi_2 \mathrm{d}t - m_f V^2 \varphi_1 \mathrm{d}t) = m_f V^2(\varphi_2 - \varphi_1). \tag{5}$$

The translational equilibrium of the first pipe gives the transversal reaction in the joint A

$$\begin{aligned} T_A &= F + T_B + \int_0^a (f_p^{(1)} + f_f^{(1)})\,\mathrm{d}x_1 + \int_0^{l-\langle l-a\rangle} f_w^{(1)}\,\mathrm{d}x_1 \\ &= F + T_B + (m_p + m_f)\ddot{\varphi}_1 \frac{a^2}{2} + 2m_f V\dot{\varphi}_1 a + k_w \frac{\varphi_1}{2}(l - \langle l-a\rangle)^2, \end{aligned} \tag{6}$$

where the singularity functions have been used:

$$\langle l-a\rangle^n = \begin{cases} 0 & \text{for } 0 \le l \le a, \\ (l-a)^n & \text{for } a < l \le a+b. \end{cases} \tag{7}$$

In fact, if the partial foundation extends over the first pipe, naturally the integration in the fourth term should extend over the entire length of the first pipe; in this case $l - \langle l-a\rangle \equiv l - (l-a)$. On the other hand, if the length of the partial foundation l is less than the length a of the first pipe, the integral should extend only until $x_1 \le l$. In this case $l - \langle l-a\rangle \equiv l$. As we see, by adopting the upper limit as $l - \langle l-a\rangle$, this formulation covers both sub-cases. The translational equilibrium of the second pipe gives the transverse reaction in the joint B:

$$\begin{aligned} T_B &= \int_0^b (f_p^{(2)} + f_f^{(2)})\,\mathrm{d}x_2 + \int_0^{\langle l-a\rangle} f_w^{(2)}\,\mathrm{d}x_2 \\ &= (m_p + m_f)\left(\ddot{\varphi}_1 ab + \ddot{\varphi}_2 \frac{b^2}{2}\right) + 2m_f V\dot{\varphi}_2 b + k_w\left(\varphi_1 a\langle l-a\rangle^1 + \frac{\varphi_2}{2}\langle l-a\rangle^2\right). \end{aligned} \tag{8}$$

The moment equilibrium of the loads acting on the first pipe with respect to the z axis passing through point A leads to

$$\begin{aligned}
&\int_0^a (f_p^{(1)} + f_f^{(1)})x_1 \, \mathrm{d}x_1 + \int_0^{l-\langle l-a\rangle} (f_w^{(1)}x_1 + m_r^{(1)}) \, \mathrm{d}x_1 + M_A - M_B + (T_B + F)a \\
&\quad = (m_f + m_p)\left[\ddot{\varphi}_1\left(\frac{a^3}{3} + a^2 b\right) + \ddot{\varphi}_2 \frac{ab^2}{2}\right] + R_A\varphi_1 + C_A\dot{\varphi}_1 - R_B(\varphi_2 - \varphi_1) \\
&\qquad - C_B(\dot{\varphi}_2 - \dot{\varphi}_1) + m_f V\dot{\varphi}_1 a^2 + m_f V^2 a(\varphi_2 - \varphi_1) + 2m_f V\dot{\varphi}_2 ab + k_r\varphi_1(l - \langle l-a\rangle^1) \\
&\qquad + k_w\left\{\varphi_1\left[\frac{(l-\langle l-a\rangle^1)^3}{3} + a^2\langle l-a\rangle\right] + \varphi_2\frac{a}{2}\langle l-a\rangle^2\right\} = 0. \qquad (9)
\end{aligned}$$

The moment equilibrium of the load acting on the second pipe with respect to the z axis passing through point B results in

$$\begin{aligned}
&\int_0^b (f_p^{(2)} + f_f^{(2)})x_2 \, \mathrm{d}x_2 + \int_0^{\langle l-a\rangle} (f_w^{(2)}x_2 + m_r^{(2)}) \, \mathrm{d}x_2 + M_B = (m_f + m_p)\left(\ddot{\varphi}_1\frac{ab^2}{2} + \ddot{\varphi}_2\frac{b^3}{3}\right) \\
&\quad + R_B(\varphi_2 - \varphi_1) + C_B(\dot{\varphi}_2 - \dot{\varphi}_1) + m_f V\dot{\varphi}_2 b^2 + k_r\varphi_2\langle l-a\rangle^1 \\
&\quad + k_w\left(\varphi_1\frac{a}{2}\langle l-a\rangle^2 + \frac{\varphi_2}{3}\langle l-a\rangle^3\right) = 0. \qquad (10)
\end{aligned}$$

Equations (9) and (10) are reducible to those of Benjamin (1961a) when no foundation is present ($l = 0$). It is convenient to introduce the dimensionless quantities

$$\begin{aligned}
&\lambda = \frac{a}{b}, \quad \mu = \frac{m_f}{m_f + m_p}, \quad \gamma = \frac{C_B}{\sqrt{R_B(m_f + m_p)b^3}}, \\
&\delta_R = \frac{R_A}{R_B}, \quad \delta_C = \frac{C_A}{C_B}, \quad \tau = t\sqrt{\frac{R_B}{(m_f + m_p)b^3}}, \qquad (11) \\
&\beta = \frac{l}{b}, \quad v = V\sqrt{\frac{m_f b}{R_B}}, \quad \chi_r = \frac{k_r b}{R_B}, \quad \chi_w = \frac{k_w b^3}{R_B},
\end{aligned}$$

where λ is the length ratio, μ the mass ratio, δ_R the stiffness ratio, δ_C the damping ratio, and β the foundation ratio. With equation (11), equations (9) and (10) yield the following dimensionless equaions of motion:

$$\begin{aligned}
&\frac{\lambda}{2}\ddot{\varphi}_1 + \frac{1}{3}\ddot{\varphi}_2 - \gamma\dot{\varphi}_1 + (\sqrt{\mu}v + \gamma)\dot{\varphi}_2 + \left(\chi_w\frac{\lambda}{2}\langle\beta-\lambda\rangle^2 - 1\right)\varphi_1 \\
&\quad + \left(\frac{\chi_w}{3}\langle\beta-\lambda\rangle^3 + \chi_r\langle\beta-\lambda\rangle^1 + 1\right)\varphi_2 = 0, \\
&\left(\frac{\lambda^3}{3} + \lambda^2\right)\ddot{\varphi}_1 + \frac{\lambda}{2}\ddot{\varphi}_2 + [\gamma(\delta_C + 1) + \sqrt{\mu}v\lambda^2]\dot{\varphi}_1 + (2\sqrt{\mu}v\lambda - \gamma)\dot{\varphi}_2 \\
&\quad + \left\{\chi_r(\beta - \langle\beta-\lambda\rangle^1) + \chi_w\left[\frac{1}{3}(\beta - \langle\beta-\lambda\rangle^1)^3 + \beta^2\langle\beta-\lambda\rangle^1\right] - \lambda v^2 + \delta_K + 1\right\}\varphi_1 \\
&\quad + \left(\chi_w\frac{\lambda}{2}\langle\beta-\lambda\rangle^2 + \lambda v^2 - 1\right)\varphi_2 = 0, \qquad (12)
\end{aligned}$$

where $(\dot{\ })$ denotes the differentiation with respect to the dimensionless time τ.

2.2. Characteristic Equation

We seek a solution in the form

$$\varphi_k = A_k e^{s\tau}; \quad k = 1, 2. \tag{13}$$

Substituting equation (13) into (12) and applying the condition for the existence of a nontrivial solution ($\sum A_j^2 \neq 0$), we obtain the characteristic equation in the form

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0. \tag{14}$$

In the case when the foundation length is limited to the first pipe ($0 \leq \beta \leq \lambda$) the coefficients a_j are denoted as $a_j^{(1)}$ and have the following expressions:

$$\begin{aligned} a_0^{(1)} &= \tfrac{1}{36}\lambda^2(3 + 4\lambda), \\ a_1^{(1)} &= \tfrac{1}{3}\left[\lambda^2(1+\lambda)\sqrt{\mu}v + (1+\lambda)^3\gamma + \delta_C\gamma\right], \\ a_2^{(1)} &= \tfrac{1}{6}\lambda v^2[\lambda(-3+6\mu) - 2] + \sqrt{\mu}\gamma v[\delta_C + (1+\lambda)^2)] \\ &\quad + \tfrac{1}{9}\beta^3\chi_w + \delta_C\gamma^2 + \tfrac{1}{3}\beta\chi_r + \tfrac{1}{3}\delta_R + \tfrac{1}{3}(1+\lambda)^3, \\ a_3^{(1)} &= -\lambda\sqrt{\mu}v^3 + \sqrt{\mu}v\left[\frac{1}{3}\beta^3\chi_w + \beta\chi_r + (1+\lambda)^2 + \delta_R\right] \\ &\quad + \gamma\left(\frac{1}{3}\beta^3\chi_w + \beta\chi_r + \delta_C + \delta_R\right), \\ a_4^{(1)} &= \tfrac{1}{3}\beta^3\chi_w + \beta\chi_r + \delta_R. \end{aligned} \tag{15}$$

If the elastic foundation is also attached, partially or fully, to the second pipe ($\beta > \lambda$), the coefficients of equation (14) are denoted as $a_j^{(2)}$; they read

$$\begin{aligned} a_0^{(2)} &= a_0^{(1)}, \\ a_1^{(2)} &= a_1^{(1)}, \\ a_2^{(2)} &= a_2^{(1)} + \tfrac{1}{18}(\beta-\lambda)\{6\chi_r(\lambda^3 + 3\lambda^2 - 1) + \chi_w(\beta-\lambda)[2\beta(\lambda^3 + 3\lambda^2 - 1) - \lambda(2\lambda^3 + 6\lambda^2 + 9\lambda + 4)]\}, \\ a_3^{(2)} &= a_3^{(1)} + \tfrac{1}{3}(\beta-\lambda)[v(\lambda^2 - 1)\sqrt{\mu} + \delta_C\gamma][3\chi_r + \chi_w(\beta^2 - \beta\lambda - \lambda)] - \chi_w\lambda v(\beta-\lambda)^2(\lambda+1), \\ a_4^{(2)} &= a_4^{(1)} + \chi_r(\beta-\lambda)(\delta_R - \lambda v^2 + \lambda\chi_r) + \tfrac{1}{36}\chi_w(\beta-\lambda)^2[12(\beta-\lambda)\delta_R - 6\lambda v^2(2\beta+\lambda) \\ &\quad - \chi_w\lambda^2(\beta-\lambda)(3\beta+\lambda)] + \tfrac{1}{3}\lambda\chi_r\chi_w(\beta^3 - 2\beta\lambda^2 + \lambda^3). \end{aligned} \tag{16}$$

2.3. Stability Analysis

The roots of equation (14) may be expressed in the complex form $s = p + iq$ where p stands for the dimensionless damping and q for the dimensionless oscillatory eigenfrequency; i is the imaginary unit. The system is stable when $p < 0$. The system is dynamically unstable when $p > 0$, while $q \neq 0$, while it is statically unstable when $p > 0$ and $q = 0$. The boundary of dynamic instability (flutter) is determined by the Hurwitz condition

$$a_1 a_2 a_3 - a_0 a_3^2 - a_1^2 a_4 = 0. \tag{17}$$

Equation (17) leads to a complete polynomial of sixth degree in terms of the unknown velocity v. Note that the order of the polynomial is not affected by the presence of the foundation, whereas the particularization to the case of the so called "follower" forces [$m_f = 0$ in equation (11) and $\mu = 0$ in equation (12)] makes equation (17) a linear polynomial of the unknown $v^2 = Pb/R_2$, where P is the amplitude of the follower force applied at the free end.

The limit for static instability (divergence) is given by the condition of vanishing characteristic root

$$a_4 = 0. \tag{18}$$

Note that equation (18) applies only for $a_4^{(2)} = 0$, i.e., in the case when the partial foundation extends over the joint B ($\beta > \lambda$). In fact, $a_4^{(1)}$, corresponding to the case when the elastic foundation is limited solely to the first pipe, is always positive. Thus, divergence cannot arise when the partial foundation is confined to the first pipe. Moreover, the static instability limit is unaffected by the mass ratio μ and by the damping ratio δ_C and the divergence critical velocity squared is easily obtained, as in the coefficient $a_4^{(2)}$ the term v appears only as a square; thus,

$$v_{\rm cr}^2 = \frac{36a_4^{(1)} + \chi_w(\beta-\lambda)^3[12\delta_R - \chi_w\lambda^2(3\beta+\lambda)] + 12\lambda\chi_r\chi_w(\beta^3 - 2\beta\lambda^2 + \lambda^3) + 36\chi_r(\beta-\lambda)(\delta_R + \lambda\chi_r)}{36\chi_r(\beta-\lambda)\lambda + \chi_w 6\lambda(2\beta^3 - 3\beta^2\lambda + \lambda^3)}. \tag{19}$$

In particular, if no foundation is present only flutter instability arises (Benjamin, 1961a), whereas both kinds of instability are possible when a foundation is fully attached to the pipes. Note that the critical velocity given in equation (19) approaches a finite value for $\lambda \to \infty$ only when the rotatory foundation alone is present. For example, for a fully attached purely rotatory foundation ($\chi_w = 0$ and $\beta = \lambda + 1$) equation (19) leads to

$$v_{\rm cr}^2 = \frac{\delta_R + \chi_r(1 + \delta_R + \lambda + \chi_r\lambda)}{\chi_r\lambda}, \tag{20}$$

whose limit for $\lambda \to \infty$ is the sought critical velocity $v_{\rm cr} = \sqrt{\chi_r + 1}$.

3. FULL OR NO ELASTIC FOUNDATION

Let us concentrate on the two limiting cases: (a) elastic foundations extend over the entire length of the system, or (b) no elastic foundation is present [already considered by Sugiyama & Païdoussis (1982)]. We investigate the effect of the length ratio $\lambda = l/b$ on the critical flow velocity. Figures 2 and 3 illustrate the critical velocity in the range $0{\cdot}1 \le \lambda \le 10$; a thin line is associated with flutter instability, whereas a thick line refers to divergence.

The case of a small mass ratio ($\mu = 0{\cdot}001$), characteristic of pipe conveying gas, is considered in Figure 2(a, b). Figure 2(a) shows that the presence of Winkler foundation with dimensionless modulus $\chi_w = 5$ acting alone (dashed line denoted by 2) does not change appreciably the critical velocity of the pipes without any foundation (dotted line denoted by 1). Indeed, curves 1 and 2 in Figure 2(a) practically coincide until λ reaches a value of 5; even for $\lambda = 10$ the difference is only 5·5%. Yet, the presence of a rotatory foundation with $\chi_r = 1$ (curves 3 and 4) introduces both qualitative and quantitative changes in the stability characteristics, as follows. (i) Divergence-type instability arises and governs the system behaviour for $\lambda > 1{\cdot}047$, when the rotatory foundation is the only one present (curve 3).

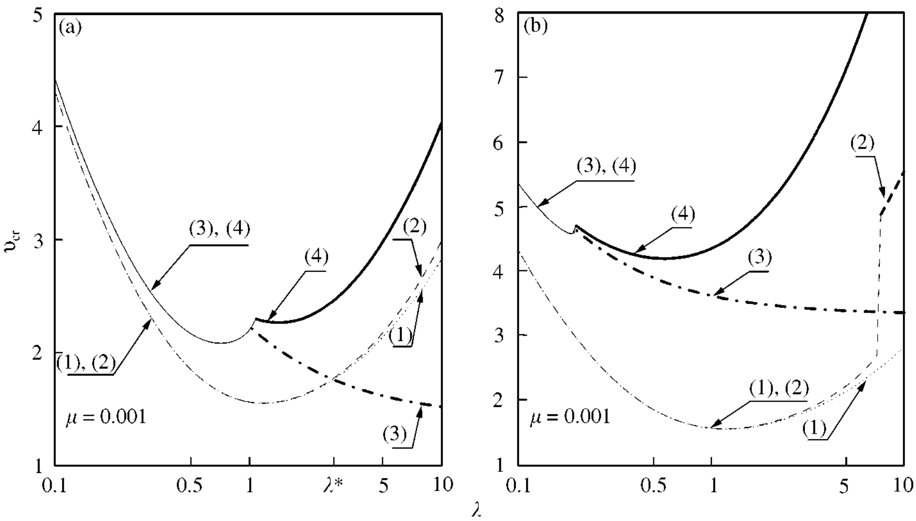

Figure 2. Dimensionless critical velocity as a function of the length ratio λ ($\gamma = 0{\cdot}001$ and $\mu = 0{\cdot}001$). Thick lines: divergence. Thin lines: flutter: (a) (1) .... $\chi_w = 0$; $\chi_r = 0$; (2) – – – – $\chi_w = 5$; $\chi_r = 0$; (3) · – · – · – $\chi_w = 0$; $\chi_r = 1$; (4) —— $\chi_w = 1$; $\chi_r = 1$ (b) (1) .... $\chi_w = 0$; $\chi_r = 0$; (2) – – – – $\chi_w = 10$; $\chi_r = 0$; (3) · – · – · – $\chi_w = 0$; $\chi_r = 10$; (4) —— $\chi_w = 10$; $\chi_r = 10$.

Note that beyond $\lambda = 1{\cdot}047$, flutter does not occur. The same happens in the region $\lambda > 1{\cdot}096$, when both kinds of foundations support the pipe with $\chi_w = \chi_r = 1$ (curve 4). Here too, beyond $\lambda = 1{\cdot}096$ flutter instability is ruled out. (ii) The minimum value of the critical velocity for which the pipe without elastic foundations undergoes flutter is $v_{cr} = 1{\cdot}547$ and is experienced for $\lambda = 1{\cdot}202$ (curve 1). Their counterparts, in the presence of rotatory foundation with $\chi_r = 1$ (curves 3 and 4) are $v_{cr} = 2{\cdot}078$ and $\lambda = 0{\cdot}724$, respectively. (iii) Increasing the length ratio, the effects due to the rotatory foundation become larger. For example, at $\lambda = 10$ the pipe without foundation loses stability through flutter at $v_{cr} = 2{\cdot}814$. Yet, when only the rotatory foundation is present, the instability occurs in the form of divergence at $v_{cr} = 1{\cdot}571$, for $\chi_r = 1$ (curve 3). When both the rotatory and the Winkler foundations ($\chi_r = 1$, $\chi_w = 1$, curve 4) are present the divergence velocity at $\lambda = 10$ is $v_{cr} = 4{\cdot}019$. Thus, elastic foundations can either decrease or increase the critical velocity associated with the foundationless system.

The case of stiffer foundation ($\chi_w = \chi_r = 10$), for the same mass ratio $\mu = 0{\cdot}001$ as in Figure 2(a), is illustrated in Figure 2(b). In this case, the Winkler foundation acting alone (dashed line, curve 2) also introduces the divergence-type instability (for $\lambda > 7{\cdot}585$). Furthermore, the transition from flutter to divergence takes place through a *jump* in the critical velocity, from $v_{cr} = 2{\cdot}692$ to $= 4{\cdot}876$. Comparing Figures 2(a) and 2(b), we note that for the case of stiffer foundation, the region ruled by divergence becomes larger.

Figure 3 portrays the system behaviour for $\mu = 0{\cdot}25$. In this setting, when any kind of foundation is present, the extent of the region governed by divergence is larger than the corresponding case with $\mu = 0{\cdot}001$.

The presence of a combined full foundation leads to an increase in the critical velocity as opposed to the foundationless system in all the analysed cases, including those for higher values of μ (not reported here). Furthermore, destabilization in the presence of a purely Winkler foundation or purely rotatory foundation occurs when such an introduction of the elastic foundation changes the instability mechanism from flutter to divergence. The purely rotatory foundation decreases the critical velocity if the length ratio $\lambda = a/b$ is beyond

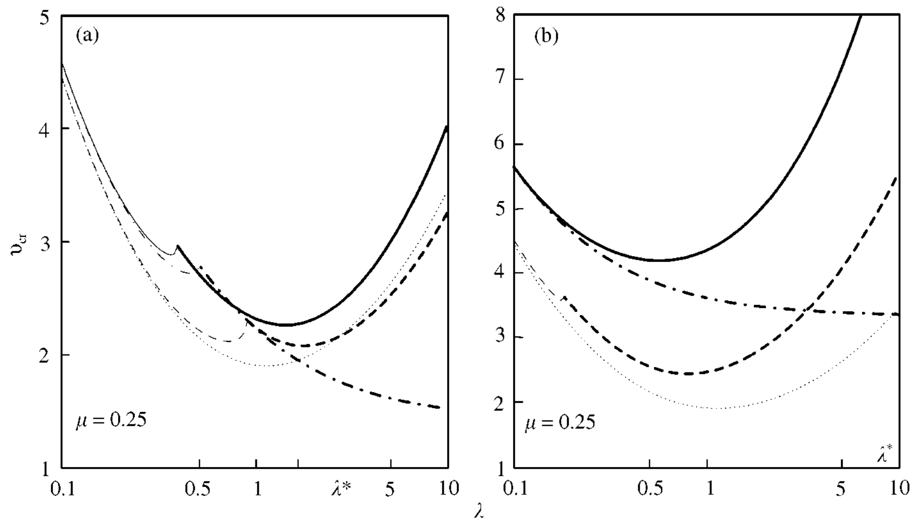

Figure 3. Dimensionless critical velocity as a function of the length ratio λ ($\gamma = 0{\cdot}001$ and $\mu = 0{\cdot}25$). Thick lines: divergence. Thin lines: flutter: (a) .... $\chi_w = 0$; $\chi_r = 0$; ---- $\chi_w = 1$; $\chi_r = 0$; $\cdot$-$\cdot$-$\cdot$- $\chi_w = 0$; $\chi_r = 1$; —— $\chi_w = 1$; $\chi_r = 1$ (b) .... $\chi_w = 0$; $\chi_r = 0$; ---- $\chi_w = 10$; $\chi_r = 0$; $\cdot$-$\cdot$-$\cdot$- $\chi_w = 0$; $\chi_r = 10$; —— $\chi_w = 10$; $\chi_r = 10$.

a certain value, λ^*. Such a transitional value λ^* is found as an intersection of curve 1 with curve 3, in Figure 2(a) corresponding to $\lambda^* = 2{\cdot}754$.

This remarkable phenomenon of having a smaller critical velocity in the system on an elastic foundation than without it is not unlike the celebrated result that was first reported byBenjamin (1961b) and by Gregory & Païdoussis (1966b). Benjamin writes (p. 493): "A remarkable phenomenon … could be observed when the pipes were freely suspended and the flow rate was large yet insufficient for instability, though being in fact considerably greater than the critical value for the simply supported case. If then the outlet end was lightly touched with a finger, the chain promptly buckled. This event was always startling to an unsuspecting observer because the system appeared quite inert before the trial and because it buckled towards the contact." Païdoussis & Li (1993) describe this unexpected pattern as follows: "A strange characteristic of this system is that, at high flow velocities but before the onset of flutter, supporting the downstream end of the cantilever by one's finger or a pencil causes if to become unstable by divergence. So, here is a case where *added support causes instability*!" (italics by Païdoussis & Li, 1993). For an interesting discussion on this paradoxical behaviour the reader may also consult with the paper by Thompson (1982). In our study too, an "unsuspecting observer" could anticipate the increase of the critical velocity due to the elastic foundations, and so it happens some times; yet it may decrease in other cases. This phenomenon takes place due to the change in the instability mechanism: a foundationless system loses its stability via flutter, while the system on the elastic foundation may lose its stability by divergence. The analogy with the works of Benjamin (1961b) and Gregory & Païdoussis (1966b) is not complete: in our case the presence of a full elastic foundation may not lead to divergence, for some combinations of the parameters (see thin line portions in Figures 2 and 3). Only in this case are the results as one would anticipate: the elastic foundation increases the critical velocity (Figures 2 and 3). Thus, the elastic foundation is associated with a richer behaviour than introduction of the elastic support (Sugiyama 1984); the latter can be represented mathematically as a product of the Winkler modulus k_w with the Dirac delta function $\delta(l)$.

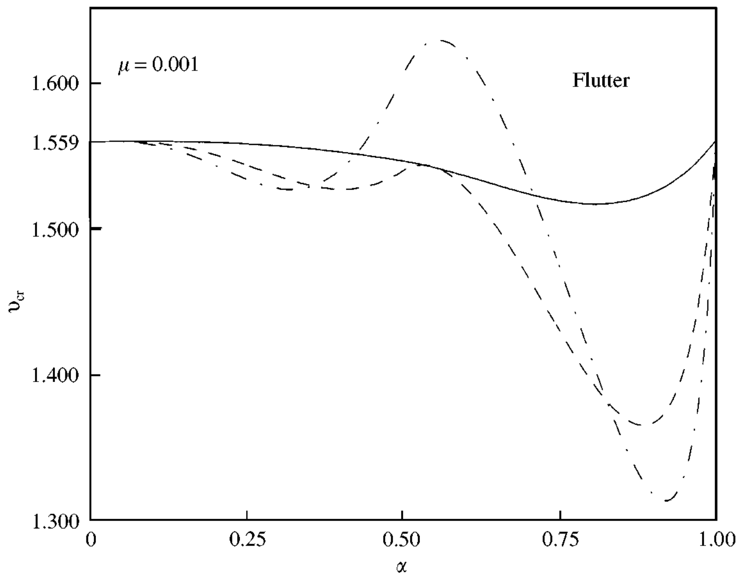

Figure 4. Pipe on purely Winkler foundation: dimensionless critical velocity as a function of the foundation attachment ratio α ($\gamma = 0{\cdot}001$, $\mu = 0{\cdot}001$): —— $\chi_w = 1$; ---- $\chi_w = 10$; ·-·-·- $\chi_w = 20$; $\alpha = 1$, $\chi_w = 1$, $v_{cr} = 1{\cdot}560$; $\alpha = 1$, $\chi_w = 10$, $v_{cr} = 1{\cdot}563$; $\alpha = 1$, $\chi_w = 20$, $v_{cr} = 1{\cdot}566$.

4. EFFECT OF A PARTIAL FOUNDATION

For a better understanding on the contribution of each type of foundation on the stability of the pipe, the cases of purely Winkler foundation as well as purely rotatory foundation will be considered first. Finally, the general case of combined foundations will be addressed. A new dimensionless variable is used for the representation of the results, namely the foundation attachment ratio $\alpha = l/(a+b)$, defined in the range [0; 1]; the zero value is associated with a system without foundation, whereas the value of unity refers to the case of fully attached foundation that we have studied in the foregoing. For specificity, we limit ourselves to pipes of the same length ($a = b$, $\lambda = 1$). Again, a thin line is associated with flutter instability, whereas a thick line refers to divergence.

4.1. Partial Winkler Foundation

The case of a Winkler foundation acting alone ($\chi_w \neq 0$, $\chi_r = 0$) is considered first. Figures 4 and 5 depict the critical velocity versus the attachment ratio α for the mass ratios $\mu = 0{\cdot}001$ and $0{\cdot}3$, respectively. The results are given for several values of the dimensionless modulus of the Winkler foundation ($\chi_w = 1$, $\chi_w = 10$, $\chi_w = 20$).

In the system with $\mu = 0{\cdot}001$ (Figure 4), only flutter instability occurs. Surprisingly, the critical velocity turns out to be a non-monotonous function of the attachment ratio, so that an increase of the region with foundation may lead to a decrease of the critical velocity. The minimum values of the critical velocity for $\chi_w = 1$ (solid line), $\chi_w = 10$ (dashed line) and $\chi_w = 20$ (dash-dotted line) are 2·8, 12·6 and 15·9%, respectively, lower than the critical velocity representing the case without foundation ($\alpha = 0$). This is another case in which a seeming strengthening of the system, by introduction of a *partial* foundation, may lead to destabilization. Moreover, contrary to the case discussed in Section 3, such a destabilization

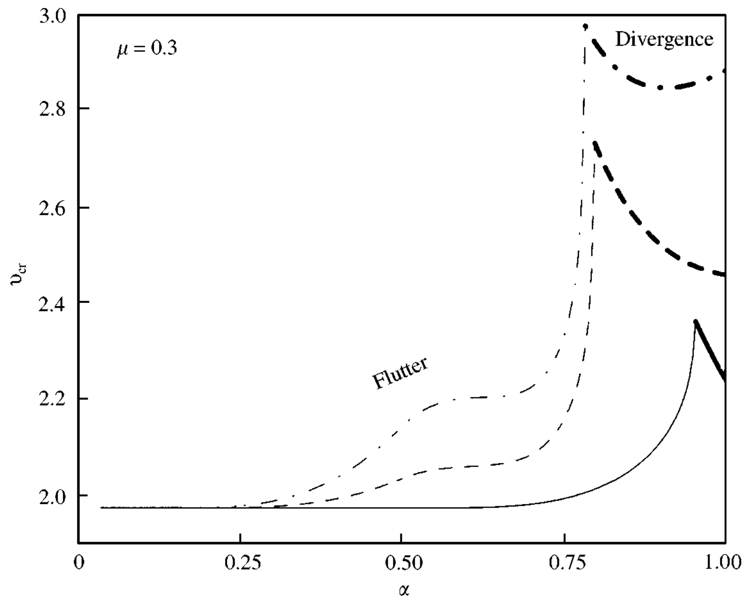

Figure 5. Pipe on purely Winkler foundation: dimensionless critical velocity as a function of the foundation attachment ratio α ($\gamma = 0{\cdot}001$, $\mu = 0{\cdot}3$). Thick line: divergence. Thin lines: flutter: —— $\chi_w = 1$; - - - - $\chi_w = 10$; ·–·–·– $\chi_w = 20$.

occurs without a *change of the instability mechanism.* Note also that the critical velocity is a nonmonotonous function of the dimensionless modulus χ_w.

Figure 5 shows that a larger mass ratio ($\mu = 0{\cdot}3$) introduces a divergence-type instability for the attachment ratios α greater than a certain value depending on the other parameters of the system. Moreover, the region in which divergence is effective expands when the nondimensional modulus χ_w increases from 1 to 20. Note that the non-monotonous behavior characteristic of the case $\mu = 0{\cdot}001$ (Figure 4) is not present anymore in the region governed by flutter.

4.2. Partial Rotatory Foundation

The critical velocities plotted in Figures 6 and 7 refer to the presence of a partial rotatory foundation ($\chi_r \neq 0$, $\chi_w = 0$). Figure 6 shows that divergence may be present also for small mass ratio ($\mu = 0{\cdot}001$), unlike the case of purely Winkler foundation (Figure 4). Moreover, the remarkable stabilizing effect due to a rotatory foundation takes place when its length passes the joint B ($\alpha > 0{\cdot}5$). The critical velocity for $\alpha = 0{\cdot}76$ and $\chi_r = 20$ is $v_{cr} = 4{\cdot}901$ (214% of the case without foundation). Nevertheless, the unexpected result of a decreasing dependence of the critical velocity on the foundation ratio is present for $\alpha < 0{\cdot}5$, $\alpha < 0{\cdot}09$ and $\alpha < 0{\cdot}05$ when $\chi_r = 1$, $\chi_r = 10$ and $\chi_r = 20$, respectively.

For a larger mass ratio, namely $\mu = 0{\cdot}3$ (Figure 7), the divergence instability regions are enhanced. In this setting, the destabilization effect is not present, although the strengthening effect in case $\chi_r = 1$, $\alpha < 0{\cdot}5$ is hardly noticeable.

4.3. Combined Partial Foundations

The stability of an articulated pipe on a generalized foundation, where the restoring forces and the restoring moments act simultaneously is of most interest. Figures 8 and 9 depict the

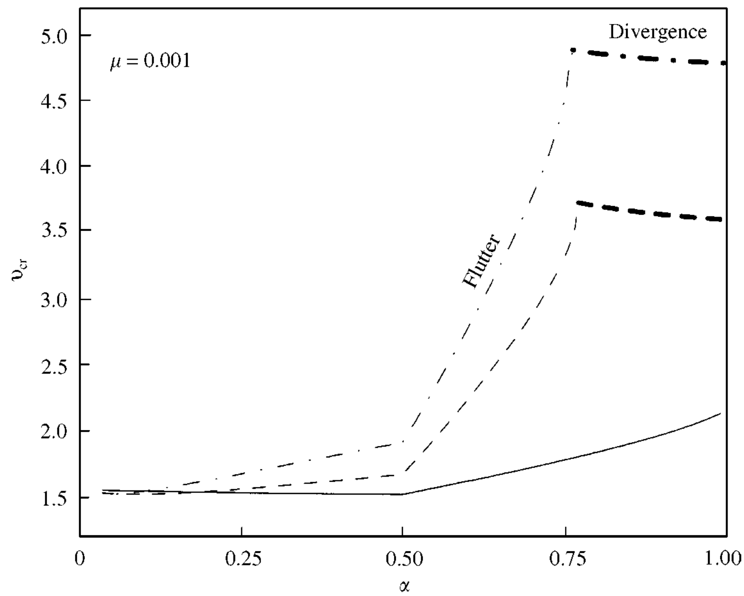

Figure 6. Pipe on purely rotatory foundation: dimensionless critical velocity as a function of the foundation attachment ratio α ($\gamma = 0{\cdot}001$, $\mu = 0{\cdot}001$). Thick line: divergence. Thin lines: flutter: —— $\chi_r = 1$; ---- $\chi_r = 10$; ·-·-·- $\chi_r = 20$.

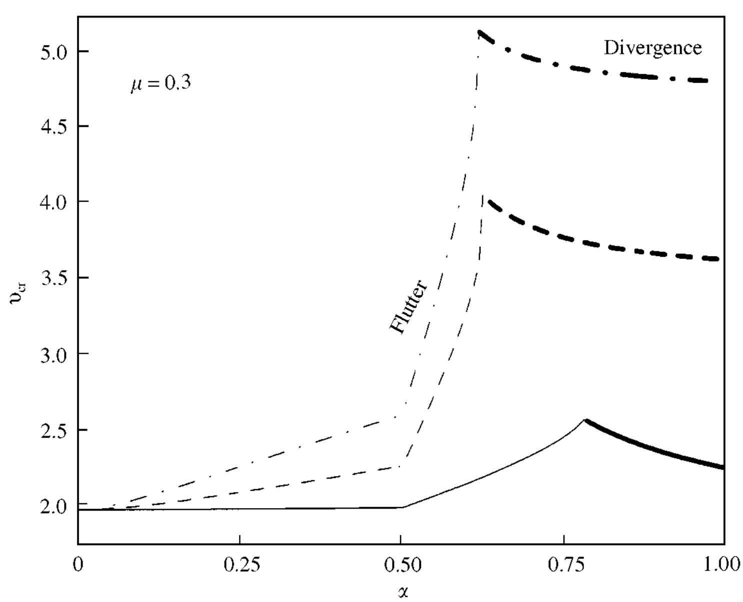

Figure 7. Pipe on purely rotatory foundation: dimensionless critical velocity as a function of the foundation attachment ratio α ($\gamma = 0{\cdot}001$, $\mu = 0{\cdot}3$). Thick line: divergence. Thin lines: flutter: —— $\chi_r = 1$; ---- $\chi_r = 10$; ·-·-·- $\chi_r = 20$.

critical velocity as a function of the foundation ratio α, for several values of the mass ratio μ, while the nondimensional foundation moduli χ_w and χ_r are fixed. The stabilizing effect of the mass ratio is evident. In the flutter region (thin lines), the non-monotonous behaviour of the critical velocity versus the foundation ratio is present for $\mu < 0{\cdot}3$, whereas for larger values

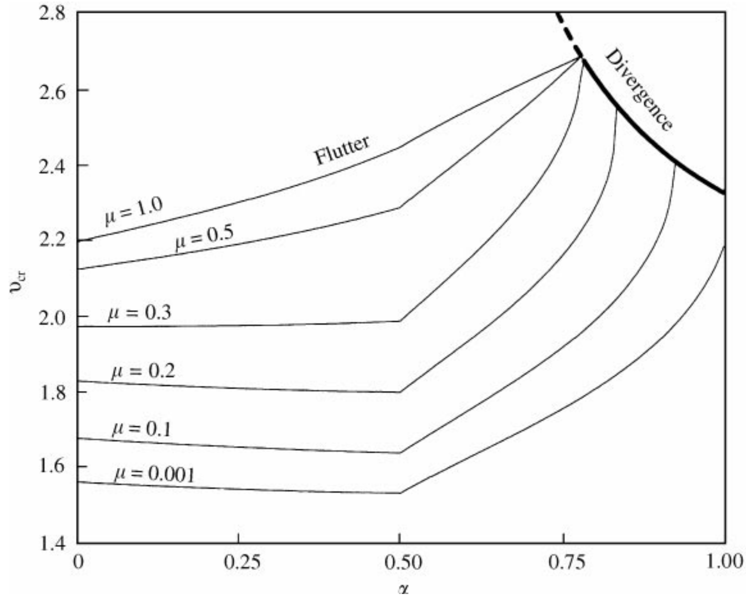

Figure 8. Pipe on combined foundation with $\chi_w = \chi_r = 1$: dimensionless critical velocity as a function of the foundation attachment ratio α ($\gamma = 0{\cdot}001$). Thick line: divergence. Thin lines: flutter.

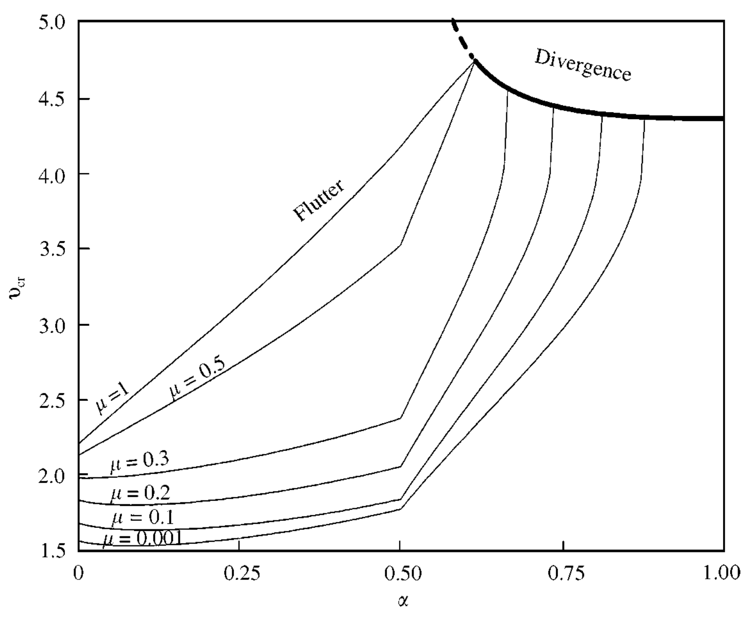

Figure 9. Pipe on combined foundation with $\chi_w = 10$, $\chi_r = 10$: dimensionless critical velocity as a function of the foundation attachment ratio α ($\gamma = 0{\cdot}001$). Thick line: divergence. Thin lines: flutter.

of μ ($\mu = 0{\cdot}5$ and 1) an increase in α leads to an increase of the critical velocity. In the divergence region (thick lines), in all the calculated settings an increment of the length of the foundation results in a decrement of the critical velocity. The maximum value of the critical velocity corresponds to the transition point between flutter and divergence instability mechanisms. As the mass ratio increases, the transition point occurs at smaller values of α, thus increasing the range of the divergence phenomenon. No divergence is present in the case of small mass ratio ($\mu = 0{\cdot}001$) associated with a small value of the nondimensional foundation moduli (Figure 8). This is in agreement with curve 2 in Figures 2(a) and 2(b).

5. CONCLUSION

We observe that the articulated pipe on partial elastic foundation, although a simple system, exhibits a rich variety of behaviours. Full elastic foundations of either kind increase the critical velocity in the case where stability is lost by flutter. Yet, if the instability arises with divergence, the critical velocity of the pipe with elastic foundations may be smaller than that in the unsupported pipe. The introduction of partial elastic foundation may destabilize the system, not unlike the previous finding by Benjamin (1961b) and Gregory and Païdoussis (1966b), where destabilization occurs by touching the pipe (without elastic foundations) with the finger. If the elastic foundation is confined within the first pipe, the system experiences flutter exclusively; yet when the attachment goes beyond the first pipe, divergence instability may occur. The change of the stability mechanism leads to interesting phenomena, illustrated by numerous results.

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